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Around King Arthur's Table

Problem Statement

King Arthur had his knights play a game to decide who would get an extra dessert, the chance to kill a dragon, and other coveted opportunities that were hot in the late 5th and early 6th centuries. He started out by placing a number on each chair around the table, starting with 1 and continuing consecutively. He would then start with the first knight, saying “you’re in,” and continued on to the second night to say, “you’re out,” and at the third knight, “you’re in.” Arthur would alternate between “you’re in” or “you’re out,” continuing around the table. If there was an empty chair he would move past it and continue the pattern. This would continue until he eliminated all but one knight. The last knight at the table was the winner.

The number of knights varied from day to day, but if you knew how many knights were going to be at the table, how could you determine which chair to sit in so you could be the winner?

Task:

Make a general rule, formula, or procedure that could predict the winning seat depending on how many knights are present.

Process:

1. We went through possible number of knights and went around the table deciding who was out and who was in until we got to the winner. We did this process from 1 to 31.
2. We then made a table of the number of knights and who won.
3. We analyzed the table and found different patterns.
4. We first found that there were different sections that correlated with the powers of 2.
5. If your looking at 2^3 the new section would start at 8 and end once you get to the 2^{4-1}-1, which is 15.
6. We then found out that if you know how many people will be there, then you can determine the winning seat by finding out what section you’re in.
7. You see what power of 2 section you’re in and start at the beginning of that section knowing that the winner of the beginning will be seat one, then count by odd consecutive numbers until you reach how many knights are in your round.

Solution:

For this problem we found two methods, both more process-based than formulaic, of finding the winning chair. The first process is as follows:

1. Count the number of seats at the table.
2. Find out in what grouping of 2^x your number of seats would fall into.
3. The product of 2^x is the number where you should begin counting the winner's seat starting with 1.
4. From there begin finding the winning chair by counting each chair with its consecutive odd integer.
5. Once you have reached the odd integer that correlates to the amount of seats to begin, sit in the chair with that number and you will win.

The second process is more mathematician-practicing-*after*-1614 friendly and formulaic:

- $2x + 1 - 2^n = y$

1. $m = log_2(x) + 1$
2. Round m down to the nearest whole number, i.e. 4.9 => 4, not 5,
3. Rounded down $m = n$
4. Put the newly found n into the $2x + 1 - 2^n = y$ equation
5. Sit in chair y and slay on

Visual Representation:

# of Knights (= x)	Knight Who Wins (= y)	Difference from # of knights and Knight who wins	Powers of 2 (exponent = n)
1	1	0	2^0
2	1	1	
3	3	0	2^1
4	1	3	
5	3	2	
6	5	1	
7	7	0	2^2
8	1	7	
9	3	6	
10	5	5	
11	7	4	
12	9	3	
13	11	2	
14	13	1	2^3

15	15	0	
16	1	15	
17	3	14	
18	5	13	
19	7	12	
20	9	11	
21	11	10	
22	13	9	
23	15	8	
24	17	7	
25	19	6	
26	21	5	
27	23	4	
28	25	3	
29	27	2	
30	29	1	
31	31	0	2^4

The colors correlate with what powers of 2 it is. (ex: 2^2 is all of the boxes highlighted orange) The powers of two refer to how many solutions there are within a section.

Evaluation:

We all enjoyed this problem! At first setting up the table and figuring out the patterns was kind of time consuming, but it made all of us think. It was hard to find the patterns that was hidden within table but once we find those hidden patterns it felt rewarding and overall very exciting. One of the hardest things for us to find was the formula/processes, which of course took time but once again we figured it out which made us all very excited and happy, though slightly frustrated that we could not find a perfect formula in time to turn it in. However, discussion with Hannah could help lead us to the correct answer, so that we have closure. Overall, this P.O.W. was both fun and challenging.

Self-Assessment:

For this P.O.W., we chose to do the write up together, with each person choosing a section to complete. It took all of us to find formulas and important details for this P.O.W., so we used both mathematical/logical skills, as well as teamwork skills. This allowed us to find a useful solution to the given problem. For both the work that we put in to this P.O.W. and for the solution we found, we all deserve a high grade.